

# Steady state entanglement between hybrid light-matter qubits

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We study the case of two polaritonic qubits localized in two separate cavities coupled by a fiber/additional cavity. We show that surprisingly enough, even a coherent classical pump in the intermediate cavity/fiber can lead to the creation of entanglement between the two ends in the steady state. The stationary nature of this entanglement and its survival under dissipation opens possibilities for its production under realistic laboratory conditions. To facilitate the verification of the entanglement in an experiment we also construct the relevant entanglement witness measurable by accessing only a few local variables of each polaritonic qubit.

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## I. INTRODUCTION

Recently, there has been a growing interest in exploiting a certain class of coupled hybrid light-matter systems, namely coupled cavity polaritonic systems, for various purposes such as for realizing schemes for quantum computation [1, 2], for communication [3] and for simulations of quantum many-body systems [4, 5, 6, 7, 8, 9, 10, 11]. These cavity-atom polaritonic excitations are different from propagating polaritonic excitations in atomic gases and exciton-photon polaritons in solid state systems [12]. This area is also distinct from those using hybrid light-matter systems in quantum computing where only the matter system (such as an atom or an electron) acts as the qubit. In the latter case the qubits are atoms and light is used exclusively as a connection bus between them [13, 14, 15, 16, 17, 18, 19]. Promising schemes to produce steady state entanglement between atoms in distinct cavities have also been proposed [19]. In these ground states of atoms have been used in order to circumvent decoherence due to spontaneous emission. In addition to auxiliary atomic levels, external driving fields as well as an unidirectional coupling between cavities are required. In polaritonic coupled cavity systems on the other hand, the localized mixed light-matter excitations, or polaritons, allow for the identification of qubits that possess the easy manipulability and measurability of atomic qubits, while also being able to naturally interact whereas separated by distances over which photons can be exchanged between them. Motivated by the rapid experimental progress in Cavity Quantum Electrodynamics and the ability to couple distinct cavities in a variety of systems [20, 21, 22, 23, 24], the realization of a system that could produce verifiable, steady state entanglement between two polaritonic qubits in currently realistic laboratory conditions would be extremely interesting. In that case the decoherence emerging from the photonic losses due to the mixed nature of the polaritons, in addition to that from atomic spontaneous emission, will need to be controlled. Therefore, apriori one may not expect a completely stationary entanglement of two polaritons unless the unavoidable loss of coherence due to both channels can somehow be “re-injected” into the system.

Here we show that even under strong dissipation in both the atomic and photonic parts, it is still possible to deterministically entangle two such polaritonic qubits. More precisely, we study the case of two polaritonic qubits coupled by a fiber/additional cavity and show that surprisingly enough, even a coherent *classical pump* can lead to the creation of entanglement between them in the steady state. The stationary nature of this entanglement should make easier its experimental verification. To this end we also provide a relevant operator (an “entanglement witness” [25]) measurable by only measuring local variables of each polariton.

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## II. THE MODEL

The Hamiltonian describing an array of  $N$  identical atom-cavity systems is the sum of the free light and dopant parts and the internal photon and dopant couplings

$$H^{free} = \omega_d \sum_{k=1}^N a_k^\dagger a_k + \omega_0 \sum_{k=1}^N |e\rangle_k \langle e|, \quad (1)$$

$$H^{int} = g \sum_{k=1}^N (a_k^\dagger |g\rangle_k \langle e| + a_k |e\rangle_k \langle g|). \quad (2)$$

Here  $a_k, a_k^\dagger$  are the photonic field operators localized in the  $k$ -th system and  $|e\rangle_k, |g\rangle_k$  are the excited and ground state of the dopant in the  $k$ -th system. Moreover,  $g$  is the light-atom coupling strength and  $\omega_d(\omega_0)$  the photonic(atomic) frequencies respectively ( $\hbar = 1$  throughout the paper). The  $H^{free} + H^{int}$  Hamiltonian can be diagonalized in a basis of mixed photonic and atomic excitations, called *polaritons*. On resonance between atom and cavity, the polaritons are created by operators  $P_k^{(\pm, n)\dagger} = |n\pm\rangle_k \langle g, 0|$ . The states  $|n\pm\rangle_k = (|g, n\rangle_k \pm |e, n-1\rangle_k)/\sqrt{2}$  are the polaritonic states (also known as dressed states) with energies  $E_n^\pm = n\omega_d \pm g\sqrt{n}$  and  $|n\rangle_k$  denotes the  $n$ -photon Fock state of the  $k$ -th cavity.

It has been shown that in an array of these atom-cavity systems the addition of a hopping photon term  $\propto \sum_j (a_j^\dagger a_{j+1} + a_j a_{j+1}^\dagger)$ , leads to a polaritonic Mott phase where a maximum of one excitation per site is allowed [5]. This originates from the repulsion due to the photon blockade effect [21]. In this Mott phase, the system's Hamiltonian in the interaction picture results

$$H_I = J \sum_k \left( P_k^{(-,1)\dagger} P_{k+1}^{(-,1)} + P_k^{(-,1)} P_{k+1}^{(-,1)\dagger} \right), \quad (3)$$

where  $J$  is the coupling due to photon hopping from cavity to cavity. Since double or more occupancy of the sites is prohibited, one can identify  $P_k^{(-,1)\dagger}$  with  $\sigma_k^\dagger = \sigma_k^x + i\sigma_k^y$ , where  $\sigma_k^x, \sigma_k^y$  and  $\sigma_k^z$  stand for the usual Pauli operators. The system's Hamiltonian then becomes the standard  $XY$  model of interacting spin qubits with spin up/down corresponding to the presence/absence of a polariton [5].

Let us now consider a linear chain of three coupled cavities with the two extremal ones doped with a two level system as shown in Fig.1(a). Alternatively, as the central cavity in any case is undoped, one can simply replace it with an optical fiber of short length (so that the distance is greatly increased but the fiber still supports a single mode of frequency near those of the two cavities), which simplifies the setting even further, as shown in Fig.1(b). For the purposes of description, we will use the three cavity setting remembering that everything applies to the case of two cavities linked by a fiber. The fact that a classical field can drive (i.e., pump energy into) the central cavity in a three cavity setting (as also shown in Fig.1(a)) is replaced in the fiber setting by a coupler feeding light into the cavity (as also shown in Fig.1(b)).

Let  $\sigma_j^\dagger = |1-\rangle_j \langle g, 0|$  be the polaritonic spin operators for the end cavities (the index  $j = 1, 2$  labels the two end cavities) and  $a, a^\dagger$  the field operators of the central empty cavity. Since the latter is not doped, there the field operators play the role of polariton operators and they couple to polariton operators of the ends cavities. Moreover, assuming that the central cavity (or fiber) is driven, the Hamiltonian describing the system dynamics will be

$$H = J \sum_{j=1}^2 \left( \sigma_j a^\dagger + \sigma_j^\dagger a \right) - \Delta a^\dagger a + \alpha a^\dagger + \alpha^* a, \quad (4)$$

where  $\Delta = \omega_{mid} - \omega_{pol}$  is the detuning between the central cavity mode of frequency  $\omega_{mid}$  and the polaritons frequency  $\omega_{pol} = \omega_0 - g$ . Furthermore,  $\alpha$  is the product of the coupling of the driving field to the central cavity field (say  $G$ ) and the amplitude of the driving radiation field (say  $\tilde{\alpha}$ ). We also assume that  $\Delta$  is much smaller than the atom-light coupling in each of the outer cavities, so that only the ground level  $|\tilde{g}\rangle = |g, 0\rangle$  and first excited level  $|\tilde{e}\rangle = (|g, 1\rangle - |e, 0\rangle)/\sqrt{2}$  of the polaritons are involved (i.e., the polaritons are still good as qubits).

Suppose that the polaritons decay with the same rate  $\gamma$  (this is the effective decay rate of the polariton due to both the decay of the cavity field and the atomic excited state), and the cavity radiation mode with rate  $\kappa$ . The quantum Langevin equations describing the dynamics will be [26]

$$\dot{\sigma}_j = iJ a \sigma_j^z - \gamma \sigma_j + \sqrt{2\gamma} \sigma_j^{in}, \quad j = 1, 2 \quad (5)$$

$$\dot{a} = i\Delta a - iJ(\sigma_1 + \sigma_2) - i\alpha - \kappa a + \sqrt{2\kappa} a^{in}, \quad (6)$$

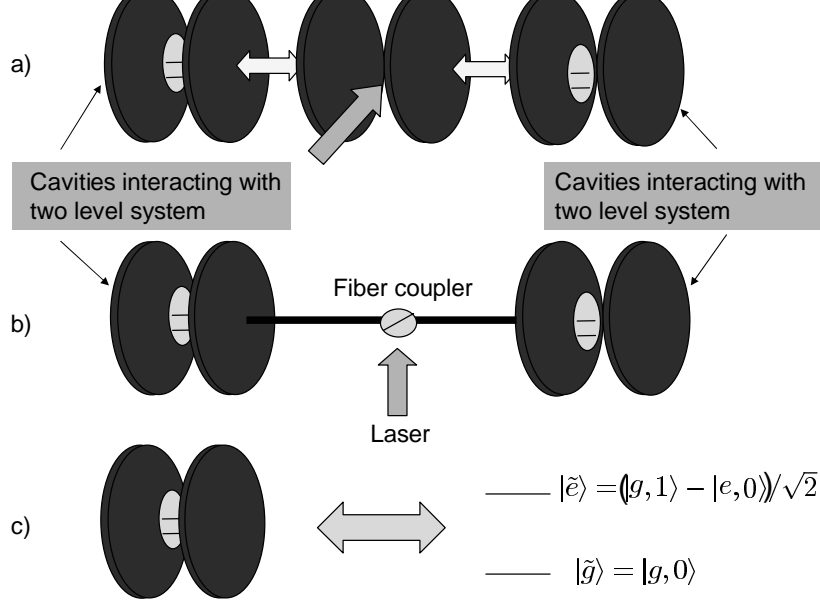


FIG. 1: The system under consideration. a) The cavities are coupled through direct photon hopping. b) The cavities are coupled through a fiber. The extremal cavities in each configuration are interacting with a two level system that could be an atom or a quantum dot depending the implementation technology used. c) The photon blockade allows for the ground and first dressed states of each atom-cavity system to be treated as a (polaritonic) qubit.

where the superscript *in* denotes the vacuum noise operators.

If  $\kappa \gg J$  the radiation mode can be adiabatically eliminated in such a way that

$$a \approx \frac{J}{\Delta + i\kappa} (\sigma_1 + \sigma_2) + \frac{\alpha}{\Delta + i\kappa} + i \frac{\sqrt{2\kappa}}{\Delta + i\kappa} a^{in}. \quad (7)$$

Moreover, if the quantities  $J/(2\sqrt{\kappa})$  and  $\alpha/(2\sqrt{\kappa})$  are large compared to the amplitude standard deviation of the fluctuating vacuum field, the last term in Eq.(7) can be neglected and

$$a \approx \frac{J}{\Delta + i\kappa} (\sigma_1 + \sigma_2) + \frac{\alpha}{\Delta + i\kappa}. \quad (8)$$

Inserting Eq.(8) into Eqs.(5), we get[29]

$$\dot{\sigma}_1 = i \frac{J^2}{\Delta + i\kappa} \sigma_2 \sigma_1^z + i \frac{J\alpha}{\Delta + i\kappa} \sigma_1^z - \gamma \sigma_1 + \sqrt{2\gamma} \sigma_1^{in}, \quad (9)$$

$$\dot{\sigma}_2 = i \frac{J^2}{\Delta + i\kappa} \sigma_1 \sigma_2^z + i \frac{J\alpha}{\Delta + i\kappa} \sigma_2^z - \gamma \sigma_2 + \sqrt{2\gamma} \sigma_2^{in}, \quad (10)$$

corresponding to an effective Hamiltonian for polaritons of the type

$$H_{eff} = \Re \left[ \frac{J^2}{\Delta + i\kappa} \right] (\sigma_1 \sigma_2^\dagger + \sigma_1^\dagger \sigma_2) + \frac{J\alpha}{\Delta + i\kappa} (\sigma_1^\dagger + \sigma_2^\dagger) + \frac{J\alpha^*}{\Delta - i\kappa} (\sigma_1 + \sigma_2). \quad (11)$$

We are using  $\Re$  and  $\Im$  to denote the real and imaginary part respectively.

The dynamics of the polaritons can now be described by the master equation [26]

$$\dot{\rho} = -i [H_{eff}, \rho] + \sum_{j=1}^2 L_j \rho L_j^\dagger - \frac{1}{2} \{ L_j^\dagger L_j, \rho \}, \quad (12)$$

where  $L_j = \sqrt{2\gamma} \sigma_j$  are the Lindblad operators.

### III. STEADY STATE ENTANGLEMENT

At the steady state Eq.(12) becomes

$$0 = -i\zeta [\sigma_1\sigma_2^\dagger + \sigma_1^\dagger\sigma_2, \rho] - i\xi [\sigma_1^\dagger + \sigma_2^\dagger, \rho] - i\xi^* [\sigma_1 + \sigma_2, \rho] \\ + 2\sigma_1\rho\sigma_1^\dagger - \sigma_1^\dagger\sigma_1\rho - \rho\sigma_1^\dagger\sigma_1 + 2\sigma_2\rho\sigma_2^\dagger - \sigma_2^\dagger\sigma_2\rho - \rho\sigma_2^\dagger\sigma_2, \quad (13)$$

where  $\zeta = \Re[J^2/\gamma(\Delta + i\kappa)]$  and  $\xi = \alpha J/\gamma(\Delta + i\kappa)$ .

The steady state solution of Eq.(13) can be found by writing the density operator and the other operators in a matrix form, in the basis  $\mathbf{B} = \{|\tilde{e}\rangle_1|\tilde{e}\rangle_2, |\tilde{g}\rangle_1|\tilde{e}\rangle_2, |\tilde{e}\rangle_1|\tilde{g}\rangle_2, |\tilde{g}\rangle_1|\tilde{g}\rangle_2\}$ . Let us parametrize the density operator as

$$\rho = \begin{pmatrix} \mathcal{A} & \mathcal{B}_1 + i\mathcal{B}_2 & \mathcal{C}_1 + i\mathcal{C}_2 & \mathcal{D}_1 + i\mathcal{D}_2 \\ \mathcal{B}_1 - i\mathcal{B}_2 & \mathcal{E} & \mathcal{F}_1 + i\mathcal{F}_2 & \mathcal{G}_1 + i\mathcal{G}_2 \\ \mathcal{C}_1 - i\mathcal{C}_2 & \mathcal{F}_1 - i\mathcal{F}_2 & \mathcal{H} & \mathcal{I}_1 + i\mathcal{I}_2 \\ \mathcal{D}_1 - i\mathcal{D}_2 & \mathcal{G}_1 - i\mathcal{G}_2 & \mathcal{I}_1 - i\mathcal{I}_2 & 1 - \mathcal{A} - \mathcal{E} - \mathcal{H} \end{pmatrix}, \quad (14)$$

where the matrix elements also respect the requirement that  $\text{Tr}\{\rho\} = 1$ . The matrix representation of the other operators comes from

$$\sigma_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \quad (15)$$

By using matrices (14) and (15) in the r.h.s. of Eq.(13), we get a single complex matrix  $M$  which must be equal to zero. Then, equating to zero the entries of  $M$  we get a set of equation for the entries of  $\rho$ . Since  $M$  is Hermitian we can consider

$$M_{jj} = 0, \quad j, k = 1, 2, 3, 4 \quad (16)$$

$$\Re\{M_{jk}\} = 0, \quad k > j \quad (17)$$

$$\Im\{M_{jk}\} = 0, \quad k > j \quad (18)$$

so to have a set of 16 linear equations. They are not all independent because of the 15 unknown parameters  $(\mathcal{A}, \mathcal{B}_1, \mathcal{B}_2, \mathcal{C}_1, \mathcal{C}_2, \mathcal{D}_1, \mathcal{D}_2, \mathcal{E}, \mathcal{F}_1, \mathcal{F}_2, \mathcal{G}_1, \mathcal{G}_2, \mathcal{H}, \mathcal{I}_1, \mathcal{I}_2)$ . Explicitly the set of equations results

$$\begin{aligned} -4\mathcal{A} + 2\xi_2\mathcal{B}_1 - 2\xi_1\mathcal{B}_2 + 2\xi_2\mathcal{C}_1 - 2\xi_1\mathcal{C}_2 &= 0, \\ -\xi_2\mathcal{A} - 3\mathcal{B}_1 - \zeta\mathcal{C}_2 + \xi_2\mathcal{D}_1 - \xi_1\mathcal{D}_2 + \xi_2\mathcal{E} + \xi_2\mathcal{F}_1 - \xi_1\mathcal{F}_2 &= 0, \\ \xi_1\mathcal{A} - 3\mathcal{B}_2 + \zeta\mathcal{C}_1 + \xi_1\mathcal{D}_1 + \xi_2\mathcal{D}_2 - \xi_1\mathcal{E} - \xi_1\mathcal{F}_1 - \xi_2\mathcal{F}_2 &= 0, \\ -\xi_2\mathcal{A} - \zeta\mathcal{B}_2 - 3\mathcal{C}_1 + \xi_2\mathcal{D}_1 - \xi_1\mathcal{D}_2 + \xi_2\mathcal{F}_1 + \xi_1\mathcal{F}_2 + \xi_2\mathcal{H} &= 0, \\ \xi_1\mathcal{A} + \zeta\mathcal{B}_1 - 3\mathcal{C}_2 + \xi_1\mathcal{D}_1 + \xi_2\mathcal{D}_2 - \xi_1\mathcal{F}_1 + \xi_2\mathcal{F}_2 - \xi_1\mathcal{H} &= 0, \\ -\xi_1\mathcal{B}_2 - \xi_1\mathcal{B}_2 - \xi_2\mathcal{C}_1 - \xi_1\mathcal{C}_2 - 2\mathcal{D}_1 + \xi_2\mathcal{G}_1 + \xi_1\mathcal{G}_2 + \xi_2\mathcal{I}_1 + \xi_1\mathcal{I}_2 &= 0, \\ \xi_1\mathcal{B}_1 - \xi_2\mathcal{B}_2 + \xi_1\mathcal{C}_1 - \xi_2\mathcal{C}_2 - 2\mathcal{D}_2 - \xi_1\mathcal{G}_1 + \xi_2\mathcal{G}_2 - \xi_1\mathcal{I}_1 + \xi_2\mathcal{I}_2 &= 0, \\ 2\mathcal{A} - 2\xi_2\mathcal{B}_1 + 2\xi_1\mathcal{B}_2 - 2\mathcal{E} - 2\zeta\mathcal{F}_2 + 2\xi_2\mathcal{G}_1 - 2\xi_1\mathcal{G}_2 &= 0, \\ -\xi_2\mathcal{B}_1 + \xi_1\mathcal{B}_2 - \xi_2\mathcal{C}_1 + \xi_1\mathcal{C}_2 - 2\mathcal{F}_1 + \xi_2\mathcal{G}_1 - \xi_1\mathcal{G}_2 + \xi_2\mathcal{I}_1 - \xi_1\mathcal{I}_2 &= 0, \\ \xi_1\mathcal{B}_1 + \xi_2\mathcal{B}_2 - \xi_1\mathcal{C}_1 - \xi_2\mathcal{C}_2 + \zeta\mathcal{E} - 2\mathcal{F}_2 + \xi_1\mathcal{G}_1 + \xi_2\mathcal{G}_2 - \zeta\mathcal{H} - \xi_1\mathcal{I}_1 - \xi_2\mathcal{I}_2 &= 0, \\ -\xi_2\mathcal{A} + 2\mathcal{C}_1 - \xi_2\mathcal{D}_1 + \xi_1\mathcal{D}_2 - 2\xi_2\mathcal{E} - \xi_2\mathcal{F}_1 - \xi_1\mathcal{F}_2 - \mathcal{G}_1 - \xi_2\mathcal{H} + \zeta\mathcal{I}_2 &= -\xi_2, \\ \xi_1\mathcal{A} + 2\mathcal{C}_2 - \xi_1\mathcal{D}_1 - \xi_2\mathcal{D}_2 + 2\xi_1\mathcal{E} + \xi_1\mathcal{F}_1 - \xi_2\mathcal{F}_2 - \mathcal{G}_2 + \xi_1\mathcal{H} - \zeta\mathcal{I}_1 &= \xi_1, \\ 2\mathcal{A} - 2\xi_2\mathcal{C}_1 + 2\xi_1\mathcal{C}_2 + 2\zeta\mathcal{F}_2 - 2\mathcal{H} + 2\xi_2\mathcal{I}_1 - 2\xi_1\mathcal{I}_2 &= 0, \\ -\xi_2\mathcal{A} + 2\mathcal{B}_1 - \xi_2\mathcal{D}_1 + \xi_1\mathcal{D}_2 - \xi_2\mathcal{E} - \xi_2\mathcal{F}_1 + \xi_1\mathcal{F}_2 + \zeta\mathcal{G}_2 - 2\xi_2\mathcal{H} - \mathcal{I}_1 &= -\xi_2, \\ \xi_1\mathcal{A} + 2\mathcal{B}_2 - \xi_1\mathcal{D}_1 - \xi_2\mathcal{D}_2 + \xi_1\mathcal{E} + \xi_1\mathcal{F}_1 + \xi_2\mathcal{F}_2 - \zeta\mathcal{G}_1 + 2\xi_1\mathcal{H} - \mathcal{I}_2 &= \xi_1, \\ 2\mathcal{E} - 2\xi_2\mathcal{G}_1 + 2\xi_1\mathcal{G}_2 + 2\mathcal{H} - 2\xi_2\mathcal{I}_1 + 2\xi_1\mathcal{I}_2 &= 0, \end{aligned} \quad (19)$$

where  $\xi_1 = \Re\{\xi\}$  and  $\xi_2 = \Im\{\xi\}$ .

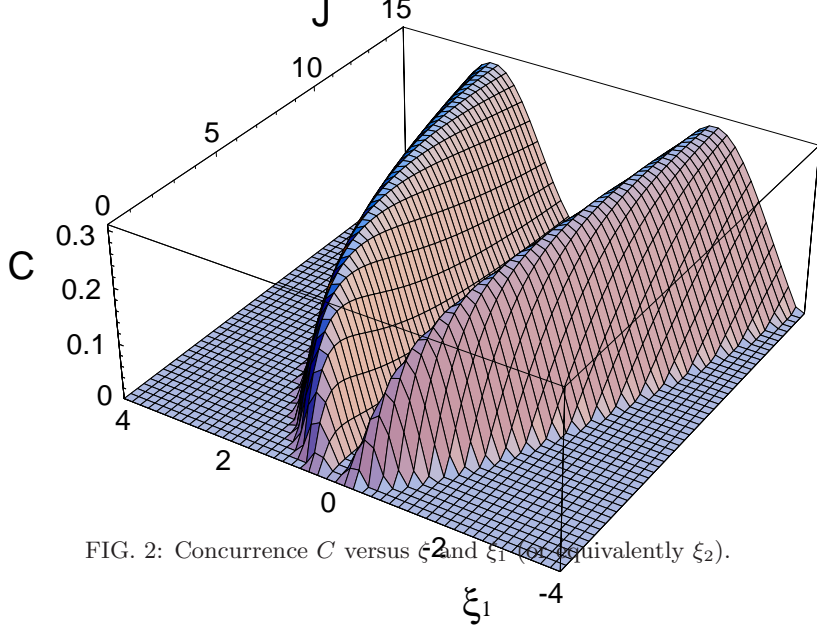


FIG. 2: Concurrence  $C$  versus  $\zeta$  and  $\xi_1$  (or equivalently  $\xi_2$ ).

Solving analytically the above set of equations we obtain for  $\xi_2 = 0$

$$\begin{aligned} \mathcal{A} &= \frac{\xi_1^4}{d}, \quad \mathcal{B}_1 = 0, \quad \mathcal{B}_2 = -\frac{\xi_1^3}{d}, \quad \mathcal{C}_1 = 0, \quad \mathcal{C}_2 = -\frac{\xi_1^3}{d}, \quad \mathcal{D}_1 = -\frac{\xi_1^2}{d}, \quad \mathcal{D}_2 = \zeta \frac{\xi_1^2}{d}, \quad \mathcal{E} = \frac{\xi_1^2 + \xi_1^4}{d}, \\ \mathcal{F}_1 &= \frac{\xi_1^2}{d}, \quad \mathcal{F}_2 = 0, \quad \mathcal{G}_1 = -\zeta \frac{\xi_1}{d}, \quad \mathcal{G}_2 = -\frac{\xi_1 + \xi_1^3}{d}, \quad \mathcal{H} = \frac{\xi_1^2 + \xi_1^4}{d}, \quad \mathcal{I}_1 = -\zeta \frac{\xi_1}{d}, \quad \mathcal{I}_2 = -\frac{\xi_1 + \xi_1^3}{d}, \end{aligned} \quad (20)$$

where

$$d = \zeta^2 + (1 + 2\xi_1^2)^2. \quad (21)$$

Notice that for  $\xi_1 = 0$  we have formally analogous solutions that lead to the same physical result, hence they are not reported.

Now that we know the stationary density matrix, we can use the concurrence as measure of the degree of entanglement [27]

$$C(\rho) = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (22)$$

where  $\lambda_i$ 's are, in decreasing order, the nonnegative square roots of the moduli of the eigenvalues of  $\rho \tilde{\rho}$  with

$$\tilde{\rho} = (\sigma_1^y \sigma_2^y) \rho^* (\sigma_1^y \sigma_2^y), \quad (23)$$

and  $\rho^*$  denotes the complex conjugate of  $\rho$ . With respect to the basis  $\mathbf{B}$  it results

$$\tilde{\rho} = \begin{pmatrix} 1 - \mathcal{A} - \mathcal{E} - \mathcal{H} & -\mathcal{I}_1 - i\mathcal{I}_2 & -\mathcal{G}_1 - i\mathcal{G}_2 & \mathcal{D}_1 + i\mathcal{D}_2 \\ -\mathcal{I}_1 + i\mathcal{I}_2 & \mathcal{H} & \mathcal{F}_1 + i\mathcal{F}_2 & -\mathcal{C}_1 - i\mathcal{C}_2 \\ -\mathcal{G}_1 + i\mathcal{G}_2 & \mathcal{F}_1 - i\mathcal{F}_2 & \mathcal{E} & -\mathcal{B}_1 - i\mathcal{B}_2 \\ \mathcal{D}_1 - i\mathcal{D}_2 & -\mathcal{C}_1 + i\mathcal{C}_2 & -\mathcal{B}_1 + i\mathcal{B}_2 & \mathcal{A} \end{pmatrix}, \quad (24)$$

In Fig.2 we show the concurrence as a function of  $\zeta$  and  $\xi_1$  (the cases  $\xi_1 = 0$  and  $\xi_2 = 0$  give the same numerical results for the concurrence). Notice that by increasing  $\zeta$ , the concurrence increases quite slowly, and a maximum amount of entanglement is approximately 0.3 for  $\zeta = 10$  and  $\xi_1 = 2.135$ . This is similar to the amount of stationary entanglement achievable with an effective interaction of the kind  $\sigma_1^z \sigma_2^z$  when combined with an intricate feedback and cascading [17].

One could try to employ entanglement witnesses to detect this entanglement [25]. A witness can be constructed from the density matrix corresponding to the maximum value of the concurrence. This would be a traceclass operator  $W$  in the Hilbert space of the two polaritonic qubits such that  $\text{Tr}[W\rho] \geq 0$  for all separable states while  $\text{Tr}[W\rho] < 0$  for the considered entangled state. The form of such a witness in the Pauli decomposition results

$$W = \sum_{j,k=id,x,y,z} c_{j,k} \sigma_1^j \otimes \sigma_2^k, \quad (25)$$

with  $\sigma^{id} = I$ . In Fig.3 we show the coefficients  $c_{j,k}$  for the entanglement witness coming from the density matrix corresponding to the maximum value of the concurrence in Fig.2. As we can see, the elements with the most

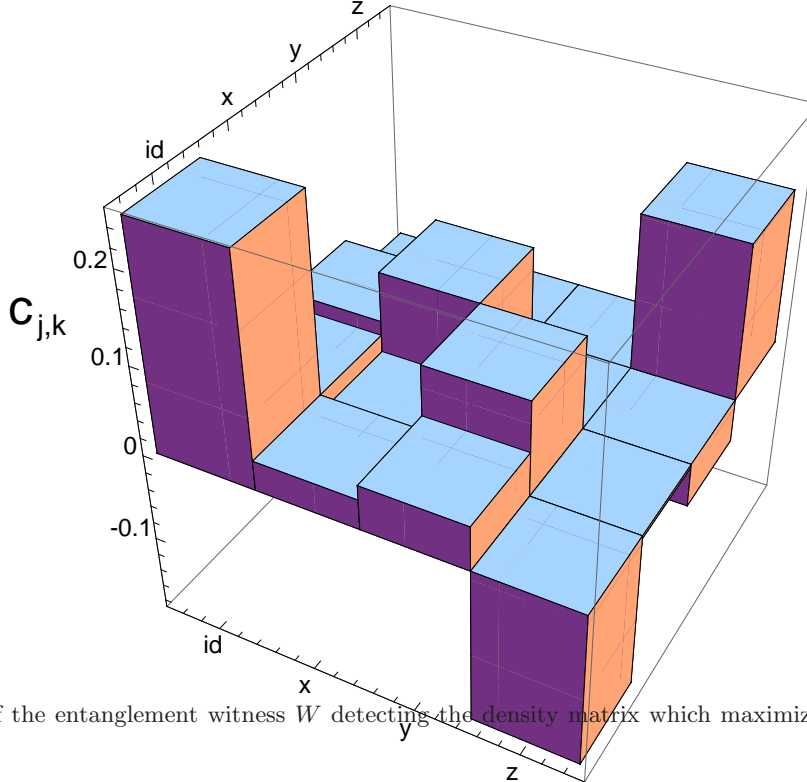


FIG. 3: Elements  $c_{j,k}$  of the entanglement witness  $W$  detecting the density matrix which maximizes the concurrence in this system.

significant weights (greater than 0.05) for measuring the witness, correspond to total of five measurements: two separate measurements of  $\sigma^z$  in each polariton, and two joint measurements  $\sigma_1^z \otimes \sigma_2^z$  and  $\sigma_1^x \otimes \sigma_2^y$ .

The values of  $\zeta$  and  $\xi_1$  used in Fig.2 to get maximal entanglement would correspond to  $\Delta = 10J$ ,  $\kappa = 10J$ ,  $G = \gamma = 0.01J$  and the pumping coherent field was also taken to have roughly a hundred photons.  $J$  is tunable and depends on the coupling of the photonic modes between neighboring cavities. Assuming this to be of the order of  $10^{10}Hz$ , this would correspond to a cavity dissipation rate  $\kappa \approx 10^{11}Hz$  and a polaritonic decay rate  $\gamma \approx 10^8Hz$ . These correspond to 0.1 nanoseconds lifetime of the cavity field and to ten nanoseconds for the polaritonic excitations at the two ends, which are within the near future in technologies like coupled toroidal microcavities and coupled superconducting qubits [22]. Coupled defect cavities in photonic crystals arrays are also fast approaching this dissipation regime and are extremely suited in fabrication of regular arrays of many coupled defect cavities interacting with quantum dots [23]. In all technologies, an increase in  $J$ , in coupling between the cavity modes, the requirements on the various lifetimes of the polaritonic and photonic field modes can be further reduced.

#### IV. CONCLUSION

To summarize, this paper presents an example of entangling two qubits in the presence of dissipation despite the fact that each qubit has a continuously decaying state. The entanglement is not transient but stationary, and thereby easy to verify in an experiment, for which there is also a relevant witness. Though the amount of entanglement is not maximal, it is still very interesting as it is for a completely open system. As opposed to the typical case of, say, many-body systems or even the case of two purely atomic qubits in a single cavity or extremely close as to be able to directly interact, here there is the added advantage that the entangled qubits are easily individually accessible (being encoded in distinct atom-cavity systems) for measurements. It is worthwhile to point out an existing scheme to have steady state entanglement between entities in distinct cavities entangles atoms [19] (as opposed to polaritons) and is much more intricate.

It is very interesting and counterintuitive that only a classical laser field driving the central cavity/connecting fiber was necessary to entangle the polaritonic qubits. A scheme feasible with current or near future technology and able to verify polaritonic entanglement as the one we have suggested in this paper, would be a significant first step towards the realization of the plethora schemes to simulate many-body systems and quantum computation using coupled cavities. Moreover, the model would also deserve to deepen counterintuitive properties of entanglement against noise (see e.g. [28]).



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